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NONLINEAR WAVES IN A MAXWELLIAN MEDIUM

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UDC 534.2+539.374

The study of propagation of nonstationary nonlinear waves in processes of explosive or shock deformation of metals involves substantial mathematical difficulties and requires, as a rule, a large expense in computer time. In many practical applications the waves generated in the metal during explosion and shock can be assumed to be weak in the sense of smallness of the relative variation of the material density in the wave [1]. Therefore it is of substantial interest to develop approximate methods of analyzing nonlinear waves, based on expanding the solutions in a small given parameter.

To solve nonlinear wave problems in hydrodynamics and elasticity theory it is presently common to develop asymptotic multiple scale methods (MSM) [2-5], making it possible to find uniformly suitable approximations to the solution of the original complex system of equations on some large time interval. The necessity of accounting for strength effects in metals upon explosive deformation or shocks with moderate velocities requires the extension of MSM to more complicated systems of equations, describing, for example, the behavior of a Maxwellian medium [6], which is elastic for small strains, and flows for sufficiently large ones. However, the application of MSM to wave problems in such media is not a formal procedure. This is related to the stress dependence of the kinetic characteristics of the medium (for example, the relaxation time of tangential stresses) in the region of the elastoplastic transition. The latter prevents direct expansion of elastoviscous terms, corresponding to the kinetics, in a series in the small parameter ε (characterizing the relative variation of the material density in the wave) from the initial condition.

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The purpose of the present study is to construct approximate equations for describing planar nonlinear deformation waves in an isotropic Maxwellian medium on the basis of multiple scale expansion techniques. Based on the suggested approach, the propagation problem is solved for a shock wave during contact explosion on the boundary of half-space.

1. In nonlinear wave theory the MSM is used to factorize the complex original system of equations into a system of independent equations for the functions being the analogs of the ordinary Riemannian invariants, i.e., constant in the zeroth approximation along their characteristic directions [4, 5]. The basis of the method is the assumption of slow variation of these functions, generated by the nonlinearity and by kinetic processes in the medium.

The original one-dimensional equations of nonlinear elasticity theory [6, 7], describing the behavior of an isotropic elastoviscous Maxwellian medium in the principal axes, are written in the Lagrangian coordinate system:

$$\frac{\partial\rho}{\partial t}+\rho\frac{\partial u}{\partial x}=0,\quad \frac{\partial u}{\partial t}-\frac{1}{\rho}\frac{\partial\sigma_1}{\partial x}=\mu_1\frac{\partial^2 u}{\partial x^2}, \tag{1.1}$$

$$\frac{\partial S}{\partial t}=-\frac{1}{2T}\varphi_k\frac{\partial E}{\partial\varepsilon_k}+\frac{1}{\rho T}\left(\mu_2\frac{\partial^2 T}{\partial x^2}+\mu_1\left(\frac{\partial u}{\partial x}\right)^2\right),$$

$$\frac{\partial\varepsilon_l}{\partial t}=\varphi_l\,(l=2,3),\quad \varphi_k=-\frac{1}{\tau(\varepsilon_i,S)}\left[\varepsilon_k-\frac{1}{2}\left(1+\frac{3\rho}{\rho_{\varepsilon_1}+\rho_{\varepsilon_2}+\rho_{\varepsilon_3}}\right)\right]$$

$$(k=1,2,3),$$

$$\sigma_1=-\rho^2E_\rho+\rho d_1E_D-\frac{1}{3}\rho\left(d_2^2+d_3^2-2d_1^2\right)E_\Delta,\quad E=E(\rho,D,\Delta,S),$$

$$\rho^2=\rho_0^2(1-2\varepsilon_1)(1-2\varepsilon_2)(1-2\varepsilon_3),\quad u=u_1,\quad u_2=u_3=0,$$

$$D=\frac{1}{2}\left(d_1^2+d_2^2+d_3^2\right),\quad \Delta=d_1d_2d_3,\quad d_i=\varepsilon_i+\varepsilon_i^2+$$

$$+\frac{1}{3}\left(-I_1+2I_2-I_1^2\right)+O(\varepsilon^3),$$

where ε_i are the principal components of the tensor of effective elastic Almans deformations. The notations in (1.1) are the commonly adopted ones. The small parameter $\varepsilon \ll 1$ of the initial condition characterizes the smallness of the relative variation of the material density in the wave, while at the same time it is understood below that the deformations correspond to a stress σ_1 exceeding the elasticity limit (for metals this pressure varies from several to dozens of GPa). It is assumed that the system (1.1) is made dimensionless by choosing as scales the uniform compression modulus $K = \rho_0c_0^2$, the initial density ρ_0, the temperature T_0, and the characteristic length scale ℓ_0 of the initial condition. Then $\mu_1 = (\zeta + {}^4/_3\eta)/\rho_0c_0\ell_0$, $\mu_2 = \kappa T_0/\ell_0\rho_0c_0^3$, where ζ and η are the viscosity coefficients of external friction, κ is the heat conduction coefficient, and the relaxation time of tangential stresses is $\tau = c_0\tau'/\ell_0$ (τ' is the dimensional value).

It is assumed that $\mu_m \ll 1$ ($m = 1, 2$) are small parameters. To apply the MSM technique to system (1.1) it is necessary to introduce, besides ε and μ_m, one more small parameter $\nu = (c_\ell^2 - c_0^2)/2c_0^2$, where c_ℓ is the phase velocity of longitudinal elastic waves of infinitely small amplitude. The relation between the small parameters is $\varepsilon \sim \mu_m \sim \nu$.

The solution is sought in the form of an expansion in small parameters

$$v_i=v_{ci}+\varepsilon v_{0i}+\varepsilon^2v_{1i}^{(\varepsilon)}+\varepsilon\mu_mv_{1i}^{(\mu_m)}+\varepsilon\nu v_{1i}^{(\nu)}+\ldots,\quad \mathbf{v}=\begin{pmatrix}\rho\\u\end{pmatrix},\quad \mathbf{v}_c=\begin{pmatrix}1\\0\end{pmatrix}. \tag{1.2}$$

The necessity of accounting for the deformation kinetics both in the elastic and in the plastic region requires retention of a substantially nonlinear shape of the dependence of the relaxation time of tangential stress τ on v_i, which prevents direct expansion of the elastoviscous terms φ_i in a series in the small parameter ε. Therefore we put $\varphi_i = \gamma(\varepsilon)\varphi_i^{(1)}\cdot(\mathbf{v}_c, \mathbf{v}_0) + \ldots$, where $\gamma \ll 1$ is a small parameter. Estimates show that in the region of elastic deformations, where $\tau = \tau_y \gg 1$, $\gamma \simeq O(\varepsilon\tau_y^{-1}) \ll \varepsilon$, while in the region of plastic deformations, where $\tau = \tau_n \ll 1$, $\gamma \simeq \varepsilon + O(\varepsilon\tau_n)$; therefore, one can put $\gamma \lesssim \varepsilon$.

Substituting (1.2) into (1.1), we find the zeroth approximation system for v_{0i}. The transition to the new unknown functions—invariants is related to diagonalization of this system, i.e., the matrix $A^{(0)}=\begin{pmatrix}0&1\\1&0\end{pmatrix}$. We introduce the matrix T_{ji} corresponding to the left eigenvectors $A^{(0)}$; then $T_{ji}A_{ik}^{(0)}T_{k\ell}=\lambda_j\delta_{j\ell}(T_{ji}T_{i\ell}=\delta_{j\ell})$, where $\lambda_{1,2}=\pm 1$ are the eigenvalues. We introduce $V_j=T_{ji}v_i=\begin{pmatrix}\rho'+u\\\rho'-u\end{pmatrix}$, where $\rho'=\rho-1$; then from (1.2) follows the expansion of V_j in small parameters, and we have for V_{0j}

$$\frac{\partial V_{0j}}{\partial t}+\lambda_j\frac{\partial V_{0j}}{\partial x}=0. \tag{1.3}$$

It is easily seen that V_{0j} are constant along their characteristic direction $dx/dt=\lambda_j$.

In passing to the following approximation we assume, according to [2], that V_{01} and V_{02} are slowly varying functions of time along their characteristic directions $V_{0j}=V_{0j}(\xi_j,t_\varepsilon,t_{\mu_1},t_{\mu_2},t_\nu)$, $\xi_j=x-\lambda_jt$, $t_\varepsilon=\varepsilon t$, $t_{\mu m}=\mu_m t$, $t_\nu=\nu t$. We then obtain for the following expansion terms

$$\frac{\partial V_{1j}^{(\varepsilon)}}{\partial t}=-\frac{\partial V_{0j}}{\partial t_\varepsilon}+\alpha_{jl}^mV_{0m}\frac{\partial V_{0l}}{\partial\xi_j},\quad \frac{\partial V_{1j}^{(\mu_1)}}{\partial t}=-\frac{\partial V_{0j}}{\partial t_{\mu_1}}-$$

$$-\frac{1}{2}(-1)^j\frac{\partial^2}{\partial\xi_j^2}(V_{01}-V_{02}),\quad \frac{\partial V_{1j}^{(\mu_2)}}{\partial t}=-\frac{\partial V_{0j}}{\partial t_{\mu_2}}+\frac{1}{2}\pi\frac{\partial^2}{\partial\xi_j^2}(V_{01}+V_{02}),$$

$$j,m,l=1,2,\quad \alpha_{jl}^m=\mathrm{const}; \tag{1.4}$$

$$\frac{\partial V_{1j}^{(\nu)}}{\partial t}=-\frac{\partial V_{0j}}{\partial t_\nu}+(-1)^j\left\{\frac{\partial V_{01}}{\partial\xi_j}+\frac{\partial V_{02}}{\partial\xi_j}-3\frac{\partial}{\partial\xi_j}\psi(V_{01}+V_{02})\right\}, \tag{1.5}$$

$$-\lambda_j\frac{\partial\psi}{\partial\xi_j}=-\{\varphi_2^{(1)}(V_{01}+V_{02},\psi)+\varphi_3^{(1)}(V_{01}+V_{02},\psi)\}=$$

$$=-\frac{\psi-\frac{1}{3}(V_{01}+V_{02})}{\tau(V_{01}+V_{02},\psi)}.$$

Here $\varepsilon\psi=-(\varepsilon_2+\varepsilon_3)$. The kinetic equation for ψ provides an implicit shape of a nonlinear operator, acting on $(V_{01}+V_{02})$ in the last term of the right-hand side of (1.5). In (1.4) $\pi=(\lambda\beta/q\rho_0)^2$ and it has also been taken into account that for adiabatic flow $\lambda_j\frac{\partial S'}{\partial\xi_j}=\mu_3\frac{1}{2}\pi^{1/2}\frac{\partial^2}{\partial\xi_j^2}(V_{01}+V_{02})+O(\varepsilon^3+\varepsilon^2\nu)$, where λ is the Lamé coefficient, β is the temperature coefficient of exchange broadening, and q is the heat capacity in the absence of stress. In writing the first equation in (1.4) it was taken into account that for $\nu\to 0$ $\varepsilon_i-\frac{1}{3}(\varepsilon_1+\varepsilon_2+\varepsilon_3)\simeq\simeq O(\varepsilon\tau_\pi)\ll\varepsilon$.

The time derivatives in the left-hand sides of (1.4), (1.5) are taken for fixed ξ_j, t_ε, $t_{\mu m}$, and in the right-hand sides $V_{0k}(\xi_j+(\lambda_j-\lambda_k),t,t_\varepsilon,t_{\mu_1},t_{\mu_2},t_\nu)$, $j\neq k$. The choice of the dependence of V_{0j} on t_ε, $t_{\mu m}$, t_ν is necessary to remove secular terms, generated by integrating (1.4), (1.5) over time t. The procedure of removing secular terms in (1.4) is standard [2]; separating in the right-hand sides of (1.4) groups of terms which are independent of t, and equating them to zero, we obtain

$$-\frac{\partial V_{0j}}{\partial t_\varepsilon}+\alpha_jV_{0j}\frac{\partial V_{0j}}{\partial\xi_j}=0;\quad -\frac{\partial V_{0j}}{\partial t_{\mu_1}}+\frac{1}{2}\frac{\partial^2V_{0j}}{\partial\xi_j^2}=0;$$

$$-\frac{\partial V_{0j}}{\partial t_{\mu_2}}+\frac{1}{2}\pi\frac{\partial^2V_{0j}}{\partial\xi_j^2}=0,\quad \alpha_j=\alpha_{jj}^j,\quad \alpha_j=(-1)^j\alpha=$$

$$=(-1)^j\frac{1}{4}\left(4+\frac{\rho_0^2}{c_0^2}(E_{\rho\rho\rho})_{\rho_0}\right),\quad j=1,2. \tag{1.6}$$

The result of integrating (1.5) is represented in the form

$$V_{1j}^{(\nu)}=\left[-\frac{\partial V_{0j}}{\partial t_\nu}+(-1)^j\left(\frac{\partial V_{0j}}{\partial \xi_j}-3\frac{\partial\psi(V_{0j})}{\partial\xi_j}\right)\right]t+(-1)^j\int_0^t\frac{\partial V_{0k}}{\partial\xi_j}dt- \tag{1.7}$$

$$-I_j^{(\nu)}(t,\xi_j,t_\varepsilon,t_{\mu_m},t_\nu),\quad I_j^\nu=(-1)^j\int_0^t 3\left(\frac{\partial\psi(V_{01}+V_{02})}{\partial\xi_j}-\frac{\partial\psi(V_{0j})}{\partial\xi_j}\right)dt,$$

$$j=1,2,\quad k\neq j.$$

Equating to zero the linear terms in t in the right-hand side of (1.7), we have

$$\frac{\partial V_{0j}}{\partial t_\nu}+(-1)^{j+1}\left(\frac{\partial V_{0j}}{\partial\xi_j}-3\frac{\partial\psi(V_{0j})}{\partial\xi_j}\right)=0,\quad -\lambda_j\frac{\partial\psi(V_{0j})}{\partial\xi_j}=-\frac{\psi(V_{0j})-\frac{1}{3}V_{0j}}{\tau(V_{0j},\psi(V_{0j}))}. \tag{1.8}$$

Equations (1.6), (1.8) guarantee the required dependence of the solutions V_{0j} on the "slow" variables t_ε, $t_{\mu m}$, t_ν. For the suggested procedure to provide a uniformly valid approximation to the solution of the exact equations (1.1) at long times, one can require finiteness of the integrals $I_j^{(\nu)}(t)$ for $t\to\infty$. It is clear from the shape of the integrals $I_j^{(\nu)}$ that this condition must be satisfied, at least for a sufficiently fast drop of the solution V_0 with the tendency $\xi_j\to\pm\infty$.

A more common condition, which must be satisfied by the integrals $I_j^{(\nu)}$, so that the procedure considered makes sense, is

$$|I_j^{(\nu)}|=O(1)\quad(j=1,2) \tag{1.9}$$

for $t\leq O[\min(\varepsilon^{-1},\ \mu_m^{-1},\ \nu^{-1})]$. The difference between the procedure suggested and the standard MSM consists in the presence of conditions (1.9), imposed on the kinetics of medium deformation. Conditions (1.9) do not guarantee global, uniform validity even with the inclusion of corrections of order ε^2, $\varepsilon\mu_m$, and $\varepsilon\nu$, but at long, though confined, times they are sufficient for the solution of the equation to be close to the exact solution.

We estimate the integrals $\tau^{-1}=\tau^{-1}H(-\sigma_1+\sigma_{1*})$ (H is the Heaviside function). After some transformations we obtain an estimate, satisfying the sufficient condition (1.9):

$$|I_j^{(\nu)}|\leqslant\left|\frac{1}{2}V_{0k\max}+\frac{\partial V_{0j}}{\partial\xi_j}\Delta t_j\right|=O(1),\quad j\neq k,\quad j,k=1,2,$$

where Δt_j is the characteristic time of loading the j-th wave, and $\Delta t_j\simeq 1$ in units of ℓ_0/c_0.

Equations (1.6) and the first of equations (1.8) are conveniently represented in the form of a single equation. Taking into account that V_{0j} is not explicitly dependent on the "fast" time t, and for fixed ξ_j $\frac{\partial}{\partial t}=\varepsilon\frac{\partial}{\partial t_\varepsilon}+\mu_m\frac{\partial}{\partial t_{\mu_m}}+\nu\frac{\partial}{\partial t_\nu}$, we have from (1.6) and (1.8)

$$\frac{\partial V_{0j}}{\partial t}-(-1)^j\varepsilon\alpha V_{0j}\frac{\partial V_{0j}}{\partial\xi_j}=\frac{1}{2}(\mu_1+\pi\mu_2)\frac{\partial^2V_{0j}}{\partial\xi_j^2}+(-1)^j\nu\left(\frac{\partial V_{0j}}{\partial\xi_j}-3\frac{\partial\psi(V_{0j})}{\partial\xi_j}\right); \tag{1.10}$$

$$-\lambda_j\frac{\partial\psi(V_{0j})}{\partial\xi_j}=-\frac{\psi(V_{0j})-\frac{1}{3}V_{0j}}{\tau(V_{0j},\psi(V_{0j}))},\quad j=1,2. \tag{1.11}$$

Thus, the solution of the system of the two independent equations (1.10) gives a uniformly valid first approximation to the solution of the exact original system of equations, at least at times $t\leq O[\min(\varepsilon^{-1},\ \mu_m^{-1},\ \nu^{-1})]$.

2. Consider the problem of a normal shock on the boundary of an isotropic half-space. From the uniformity of the initial state of the medium follows the vanishing of the function V_{02}, corresponding to a negative eigenvalue λ_2, and the absence of tangential stresses eliminates shear waves.

To study wave effects in half-space it is necessary to rewrite Eq. (1.10) for V_{01} in terms of a boundary-value problem with the replacement of $(t,\ x-\lambda_1t)$ by $(x,\ t-x/\lambda_1)$. The equation for V_{01} is represented in the form

$$\frac{\partial V_{01}}{\partial y}=\frac{1}{2}\varepsilon V_{01}\frac{\partial V_{01}}{\partial\xi}+\mu_1\frac{\partial^2V_{01}}{\partial\xi^2}+\nu\frac{\partial z}{\partial\xi}. \tag{2.1}$$

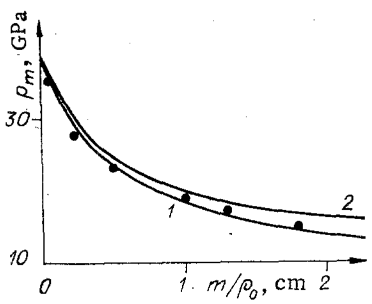

Fig. 1

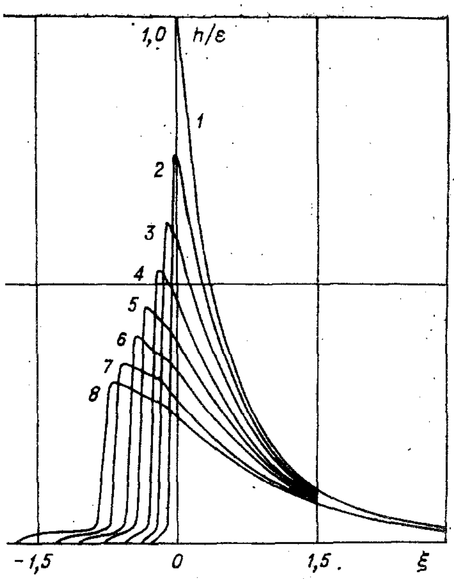

Fig. 2

Here $\xi = \omega t - y$; $y = \omega m\alpha/\rho_0 c_0$; $m = \rho_0 x$ is the mass of the Lagrange coordinate; ω^{-1} is the characteristic time from the boundary condition, $\varepsilon V_{01} = h + u/c_0$; $h = p/\rho_0 c_0^2$; p is the hydrostatic pressure; $\varepsilon = p_{\max}/\rho_0 c_0^2$; $\nu = \frac{c_l^2 - c_0^2}{2\alpha c_0^2}$; $\mu_1 = \left(\zeta + \frac{4}{3}\eta\right)\omega/2\rho_0 c_0^2$; $\alpha > 1$ is a parameter from the equation of state; $\varepsilon\nu\alpha z = -S_1/\rho_0 c_0^2$; S_1 is the component of the stress deviator, reduced to principal axes. Taking into account that in the given case $\varepsilon V_{02} = h - u/c_0 \simeq 0$ accurately up to terms of order $O(\varepsilon^2 + \varepsilon\mu_m + \varepsilon\nu)$, in (2.1) one can put $\varepsilon V_{01} \simeq 2h$. The kinetic equation for z is

$$\varepsilon \frac{\partial z}{\partial \xi} = 2\frac{\partial h}{\partial \xi} + \varepsilon\varphi(h, z) \tag{2.2}$$

(φ is the relaxation function). For a Maxwellian model of an elastoviscous medium $\varphi = -z/\tau(h, z)$. For the model of an ideal elastoplastic medium Eq. (2.2) is written as

$$\frac{1}{2}\varepsilon\frac{\partial z}{\partial \xi} = \frac{\partial h}{\partial \xi}H(h_s - h),\ \xi \leqslant \xi_m;\ \frac{1}{2}\varepsilon\frac{\partial z}{\partial \xi} = \frac{\partial h}{\partial \xi}H(h - h_m + 2h_s),\ \xi > \xi_m \tag{2.3}$$

where H is the Heaviside function, h_m is the maximum pressure in the wave profile (it corresponds to the coordinate ξ_m); $h_s = -\sigma_{1s}/\rho_0 c_l^2$; σ_{1s} is the limiting elasticity on the Hugoniot adiabat, and the first equation corresponds to the loading process while the second corresponds to the unloading process. It is easily verified that for an elastoviscous medium with $\tau^{-1} = \tau_\Pi^{-1}H(-\sigma_1 + \sigma_{1s})$, on imposing the choice $\tau_\Pi \ll 1$ from (2.2) follows the first equation of (2.3) accurately up to terms of order $O(\varepsilon\tau_\Pi)$.

It is of interest to compare the results of numerical solution of the original equations of medium flow with the solutions of the approximate equation (2.1). We examine the problem of evolution of a compression wave in iron with a contact explosion layer (without account of phase transitions) on the basis of (2.1), (2.3), while the similar problem with numerical solution of the exact original system of flow equations of an ideal elastoplastic medium was considered in [8]. The equation of state $E = (p - c_0^2(\rho - \rho_0))/(n - 1)\rho$ leads to $\alpha = (n + 1)/2$, with the original parameters for iron: $n = 5.5$, $h_s = 0.00571$, and explosive layer thickness 50/50 1.68 cm. The small parameters of the problem are: $\varepsilon = p_m'/\rho_0 c_0^2 = 0.32$ (p_m' is the pressure upon pressure decay at the contact boundary), $\nu = 0.141$, and the quantity $\mu_1 = 0.0032$ was selected by stability considerations of the numerical solution. The boundary condition h(0, t) corresponds to the analytic solution of the reflection problem of a detonation wave from a deformable wall [9].

The numerical solution of (2.1), (2.3) was carried out by the method of [10], and the results are shown in Figs. 1 and 2. Figure 1 shows the distance dependence of the shock wave amplitude: curve 1) for an elastoplastic medium; 2) the hydrodynamic approximation ($\nu = 0$); satisfactory agreement is observed with the results of numerical solution of the exact system of equations (points), obtained in [8]. The hydrodynamic approximation, as could be expected, provides an enhanced value of the pressure amplitude. This is reflected by the fact that despite the smallness of the elastic loading amplitude in comparison with the wave amplitude, it has a substantial effect on the wave profile (Fig. 2). Figure 2 also shows clearly the dynamics of formation and evolution of the elastic foreshock and load at the following distances from the boundaries $\varepsilon y = 0$, 0.206, 0.488, 0.862, 1.33, 1.9, 2.55, and 3.3 (lines 1-8).

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ION AND NEUTRAL-PARTICLE KINETICS IN A LOW-PRESSURE DISCHARGE CONTAINING A CLOSED HALL CURRENT

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UDC 533.95

Considerable interest attaches to heavy-particle kinetics in a real bounded system; there are various papers on the kinetics of neutral particles near the wall in a fusion reactor such as [1, 2]. One has to consider heavy-particle kinetics in relation to the boundary layer between a dense cold completely ionized plasma and a negative electrode [3]. As the distribution of the neutral particles near the bounding wall is spatially inhomogeneous in such cases, one has to consider the effects on the ion distribution and in particular on the ion transport in the gas from which the ions are derived. It is also important to consider heavy-particle conservation and dynamics for a low-pressure discharge containing a closed Hall current as used in generating accelerated-ion beams [4]. In that case, one cannot restrict consideration to a single component of the heavy particles. Studies have been made [5, 6] on the kinetics of neutral particles and ions in plasma accelerators with closed drift, but allowance was made only for the ionization (the system of kinetic equations for the heavy components was integrated numerically). However, these studies have neglected the interactions between the ions and the neutral particles, which can be important under certain conditions [7].

The purpose of this study is to examine the kinetics of the heavy particles in a low-pressure discharge having closed drift for the magnetized electrons, with allowance for the burnout of the neutral component because of ionization by electron impact and collisions between ions and neutral particles.

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